Differentiable Diffusion MRI Simulator for Reconstruction using Non-Learnable Parameters

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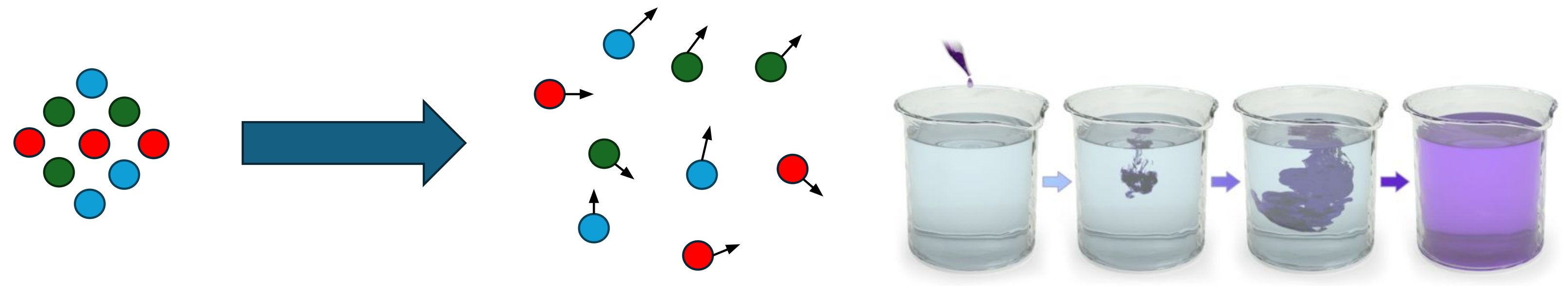
What is Diffusion and Diffusion MRI?

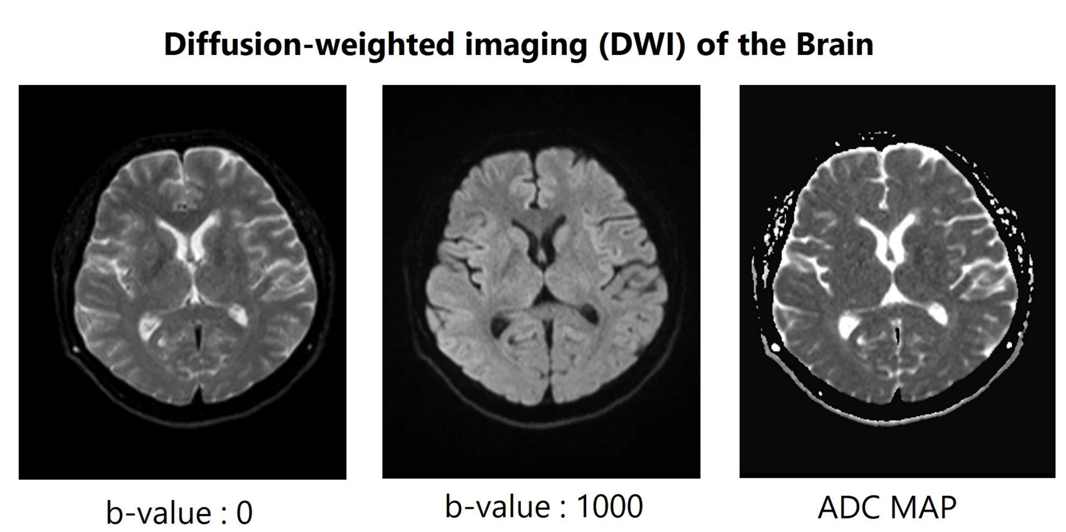
Diffusion:

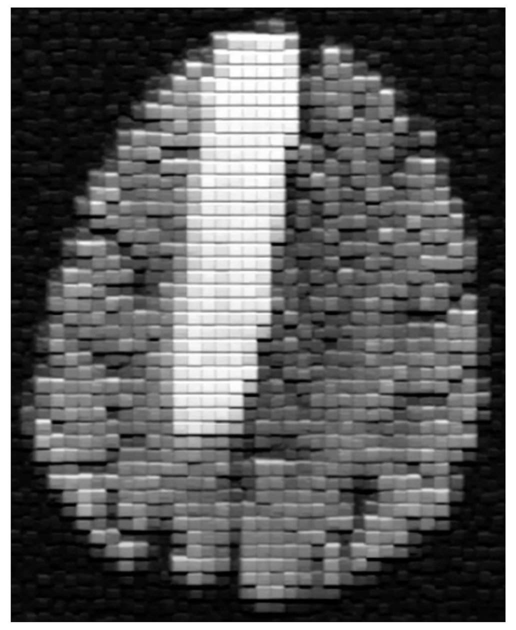
- Diffusion is the random motion of particles, for example the motion of water molecules in a medium (Brownian motion).
- Biological tissues also contain a substantial amount of water.
- Cell structure and tissue properties hinder this random motion of water.



Using Diffusion for Magnetic Resonance Imaging:

- Diffusion MRI (dMRI) is a magnetic resonance imaging technique that measures the diffusion of water molecules in biological tissues.
- This type of Magnetic Resonance Imaging is thus more sensitive to the moment of water molecules.

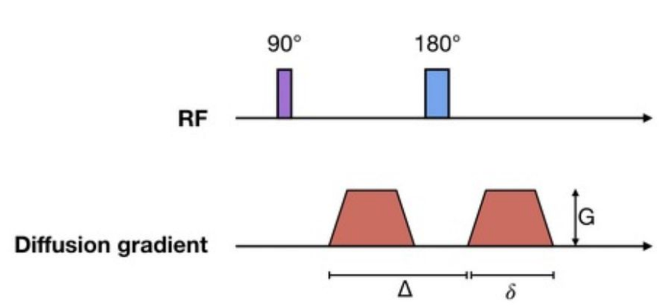


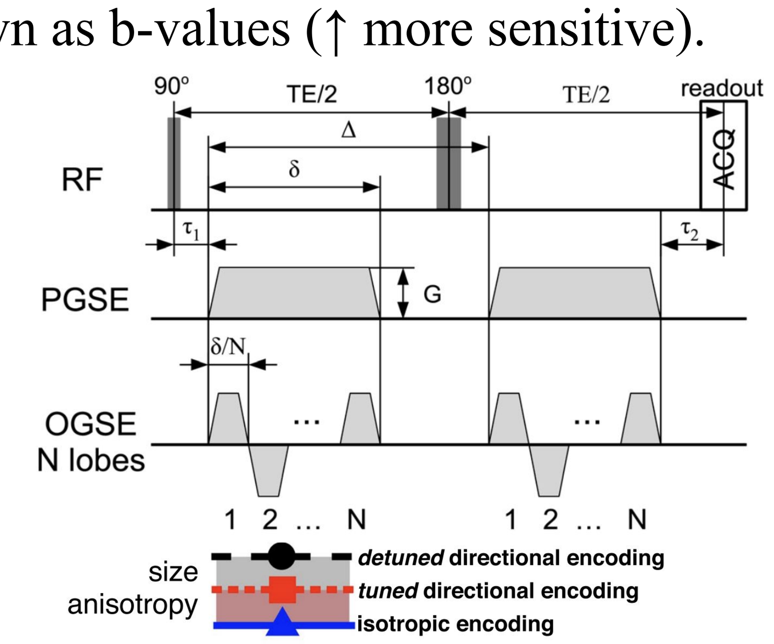


Diffusion Weighted Imaging (DWI):

- Generates an image contrast that depends on the random microscopic motion (diffusion) of water protons.
- The image gets substantially altered by different cell structures and pathological processes.
- The sensitivity the motion of water molecules can be controlled using diffusion weighting, commonly known as b-values (↑ more sensitive).

$$b = \gamma^2 G^2 \delta^2 \left(\Delta - \frac{\delta}{3}\right)$$

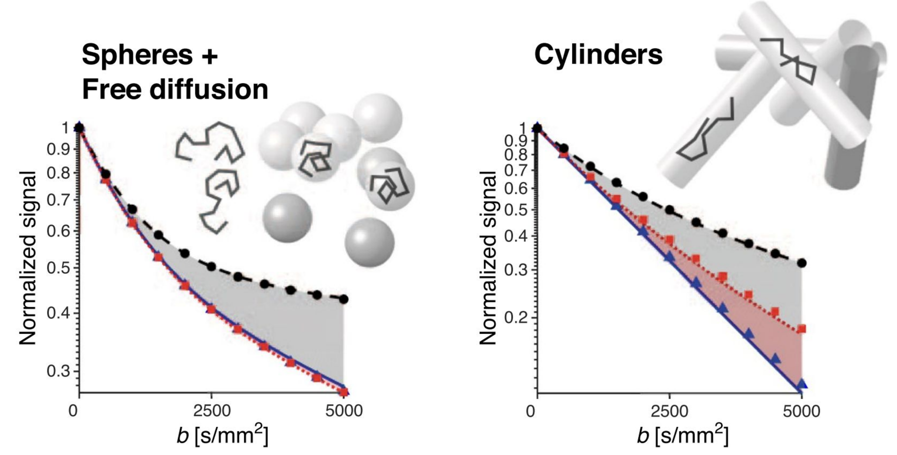




$$S_{DWI} = S_{b=0} \times e^{(-b \times D)}$$

equivalent to...

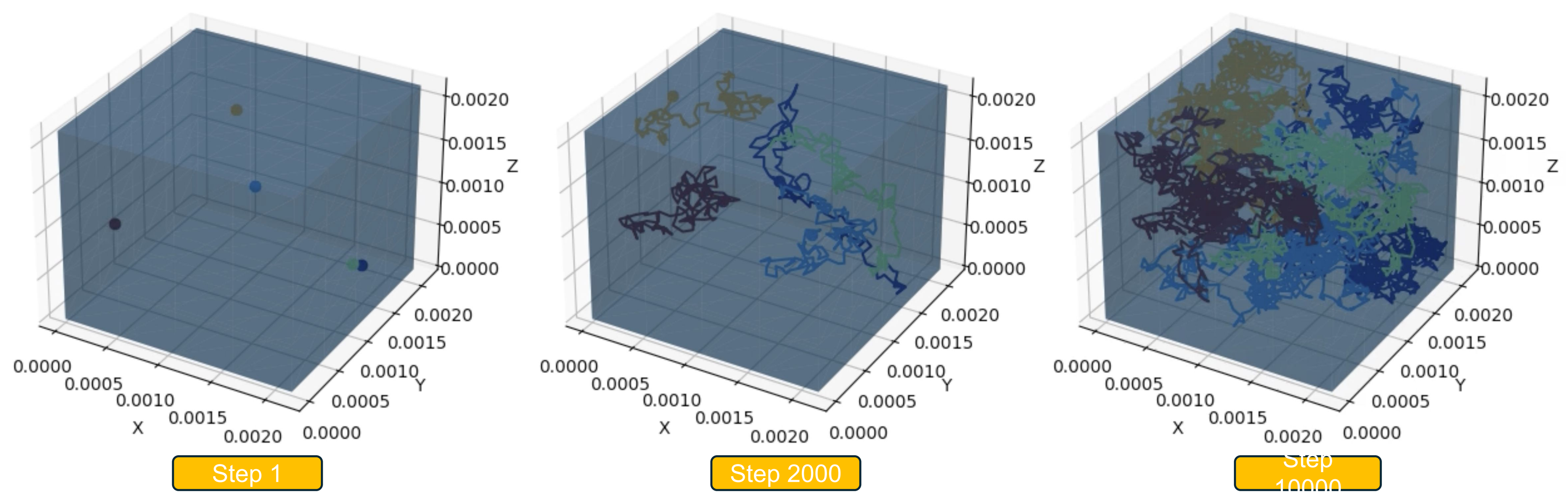
$$D = -\frac{1}{b} \times \ln\left(\frac{S_{DWI}}{S_{b=0}}\right)$$



How can we simulate Diffusion MRI?

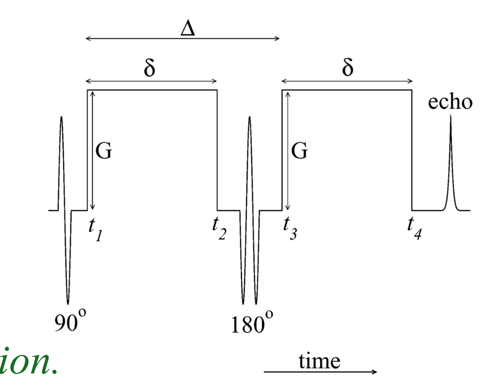
Particle based simulation or Monte-Carlo based simulations:

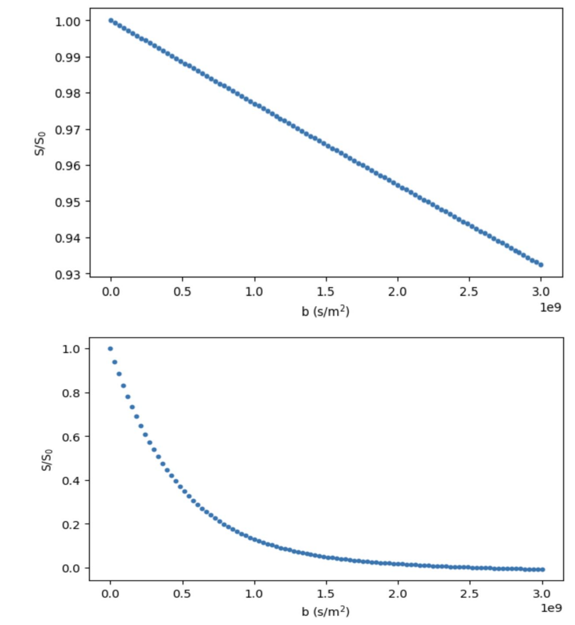
- Simulates and scans the motion (positions) of individual particles in a defined medium at fixed time steps.
- Track the total amount of phase change observed for each particle (phase accumulation) at each scan.
- Sum up the phase accumulation for all particles to obtain the signal (S).



$$d\Phi = a\gamma \mathbf{G}(t) \cdot \mathbf{x}(t) dt$$

$$S = \sum_j e^{i\Phi_j}$$





Pros:

- *Fast and Intuitive signal generation.*
- *Can work for complex shapes.*

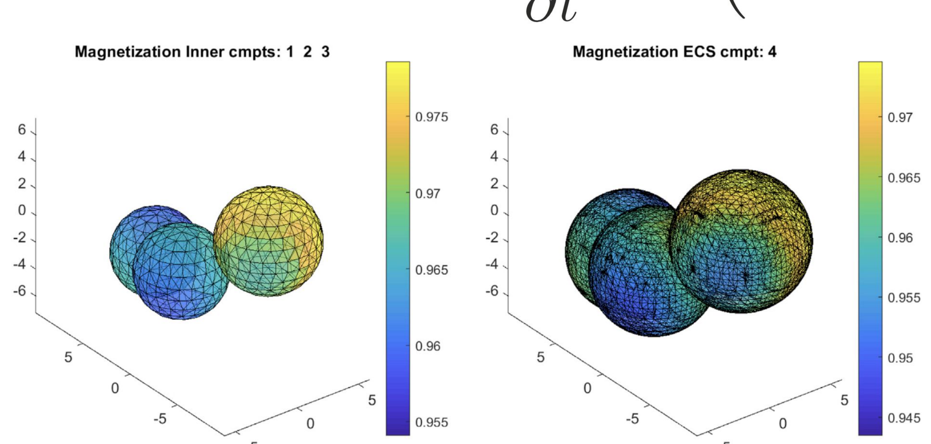
Cons:

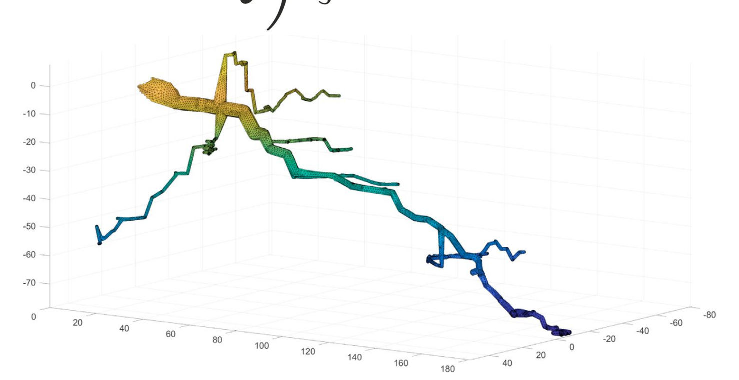
- *Memory intensive (each particle's trajectory is stored)*
- *Number of particles can be a limitation for simulation.*
- *Not differentiable (due to random sampling operations)*

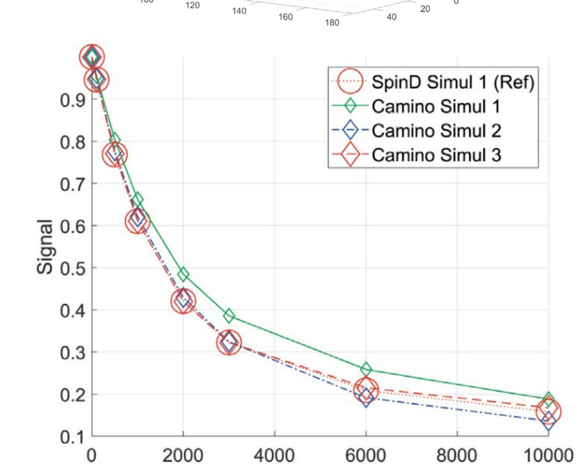
Simulation using Bloch-Torrey Equations:

- Bloch-Torrey PDEs combine Bloch equations for magnetization with diffusion terms to model spin behavior.
- These equations describe how magnetization evolves over time in the presence of magnetic fields and diffusion.
- Numerical methods solve these PDEs to simulate MRI signals in complex tissue geometries.
- Simulations can incorporate various tissue properties, gradient sequences, and diffusion characteristics for realistic modeling.

$$M\frac{\partial \xi}{\partial t} = -\left(I\gamma f(t)\, \boldsymbol{J} + \boldsymbol{S} + \boldsymbol{\underline{Q}}\right)\xi$$







Pros:

- *Can work for highly detailed and complex shapes.*
- *More reproducible.*
- *No limitation due to particles.*

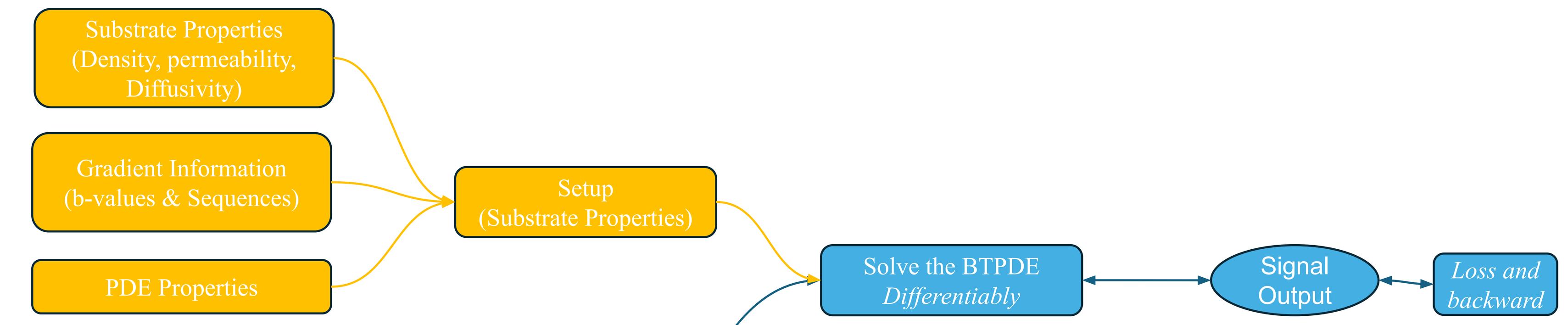
Cons:

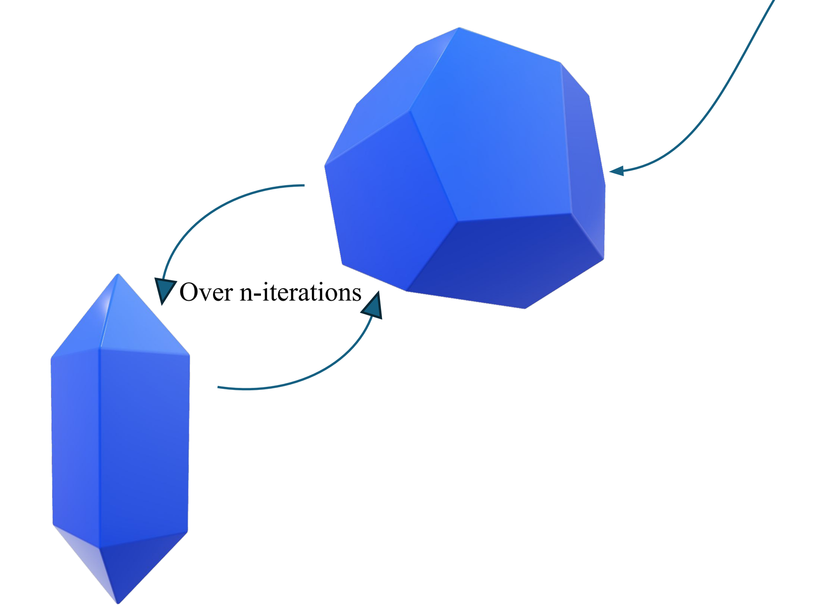
- *Slow for very complex shapes (CPU based)*
- *Not Differentiable (due to included functions).*

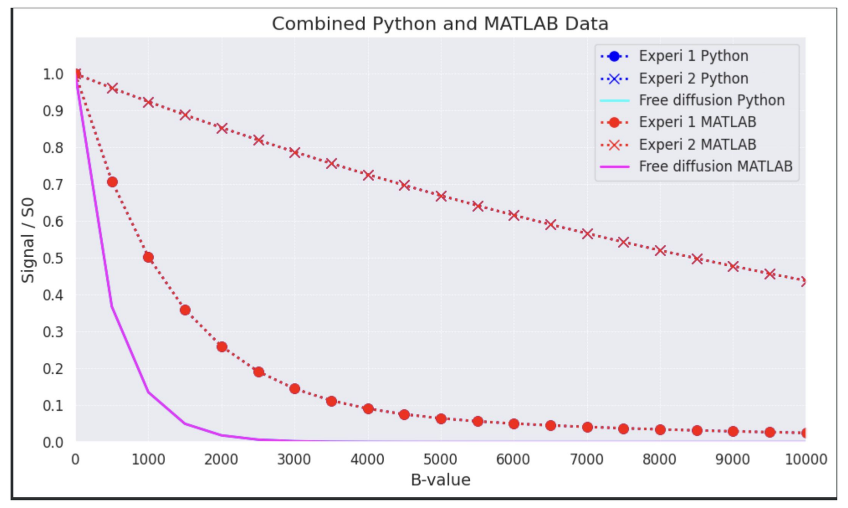
Making the Simulator Differentiable

Approach:

- We update and build from the Bloch-Torrey Equations approach.
- Each function of the Physics based simulator is written in Pytorch.
- Taking the advantage of Pytorch's Automatic Differentiation feature and GPU based parallelization.
- Using the AutoDiff and the computational graph created by Pytorch allows us to perform Back-Propagation from any point in the Simulation process.

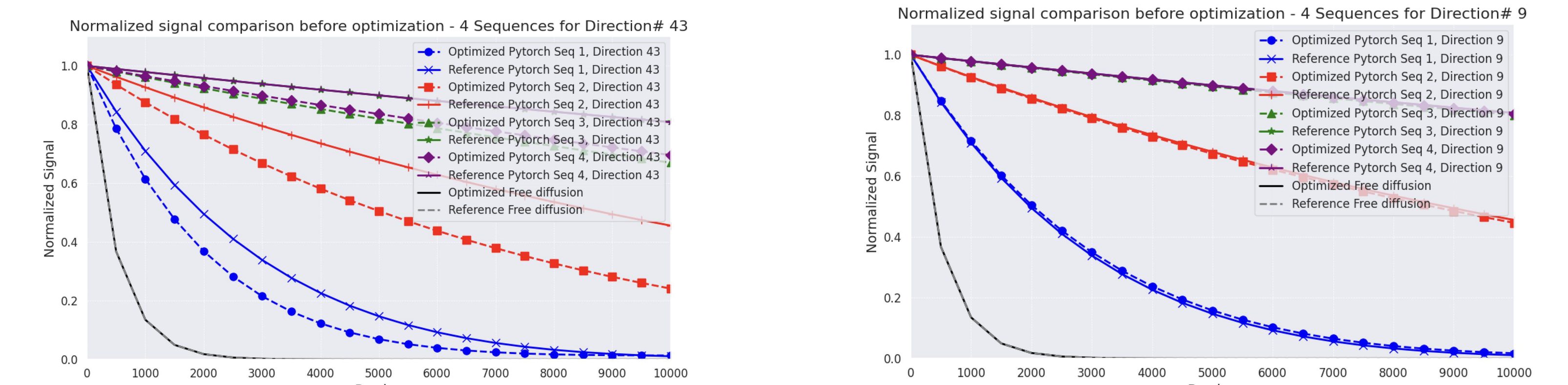






Results:

- The differentiable simulator allows for updating of the input mesh directly over iterations reconstructing the desired mesh based on reference signal.
- Allows for visualizing the shape creating the dMRI signal for a given voxel.
- Can reconstruct arbitrary meshes only using signal generated by by that mesh.



Possible improvements and future scope:

- Ease the Ill-posed nature of the current reconstruction problem.
- Use additional networks to estimate substrate parameters instead of using fixed values.

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